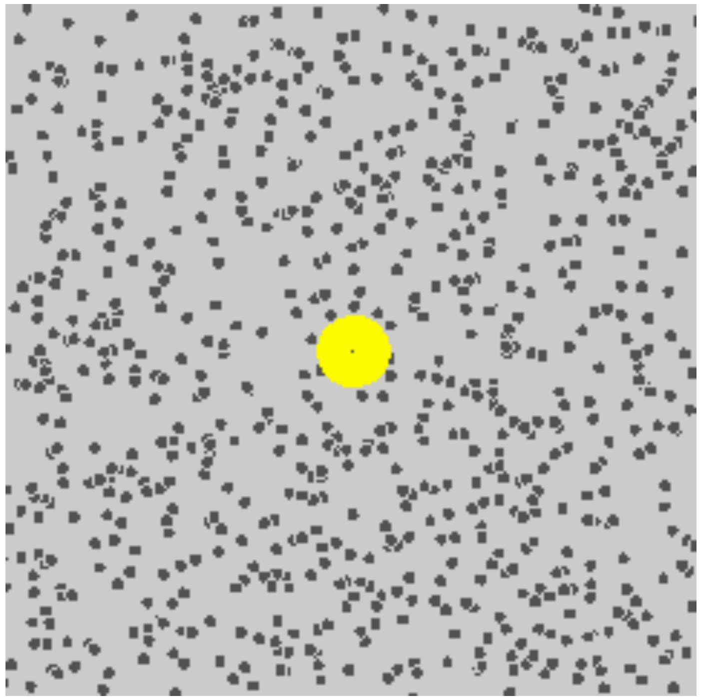
Brownian Motion Lab

JHU Advanced Lab

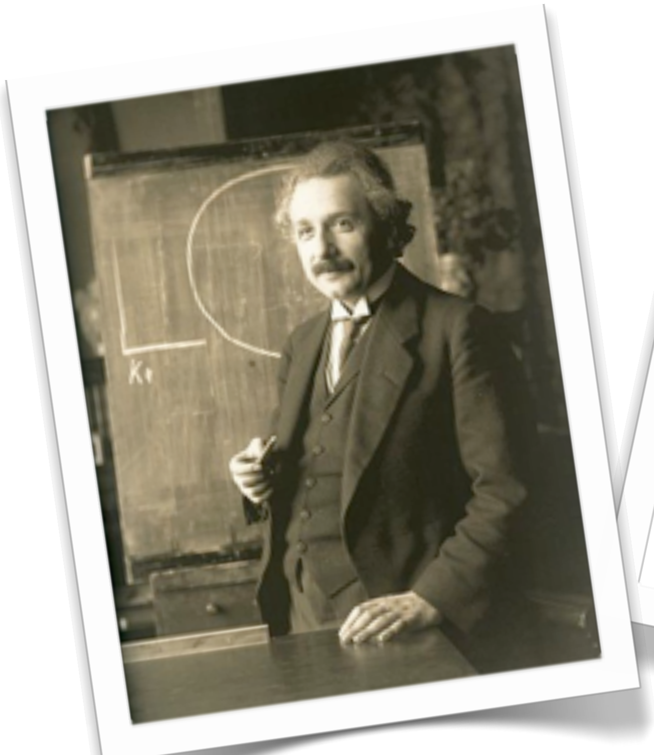


"Observe what happens when sunbeams are admitted into a building and shed light on its shadowy places. You will see a multitude of tiny particles mingling in a multitude of ways... their dancing is an actual indication of underlying movements of matter that are hidden from our sight... It originates with the atoms which move of themselves [i.e., spontaneously]. Then those small compound bodies that are least removed from the impetus of the atoms are set in motion by the impact of their invisible blows and in turn cannon against slightly larger bodies. So the movement mounts up from the atoms and gradually emerges to the level of our senses, so that those bodies are in motion that we see in sunbeams, moved by blows that remain invisible."

Lucretius c. 60 B.C.

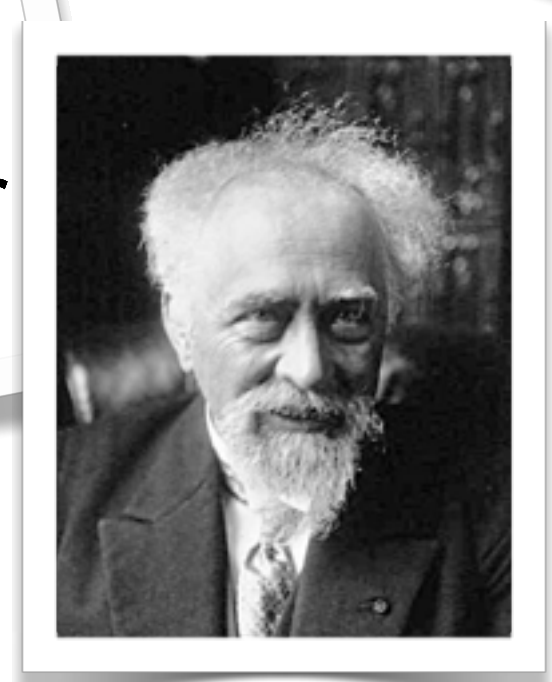
Introduction

1827 - Botanist Robert Brown (namesake) used an early microscope to observe random motion of pollen in water.



1905 - Albert Einstein publishes molecular theory of Brownian Motion. [Also Smoluchowski (1906) and Langevin (1908)]

1908 - Jean Baptiste Perrin confirms molecular theory by observing quantitative behavior of Brownian motion.



Brownian Motion establishes reality of atoms and molecules.

Theory

$$\langle x^2 \rangle = \frac{k_b T}{3\pi r \eta} t$$

Presentation Note!
Always define
all variables
in equations.

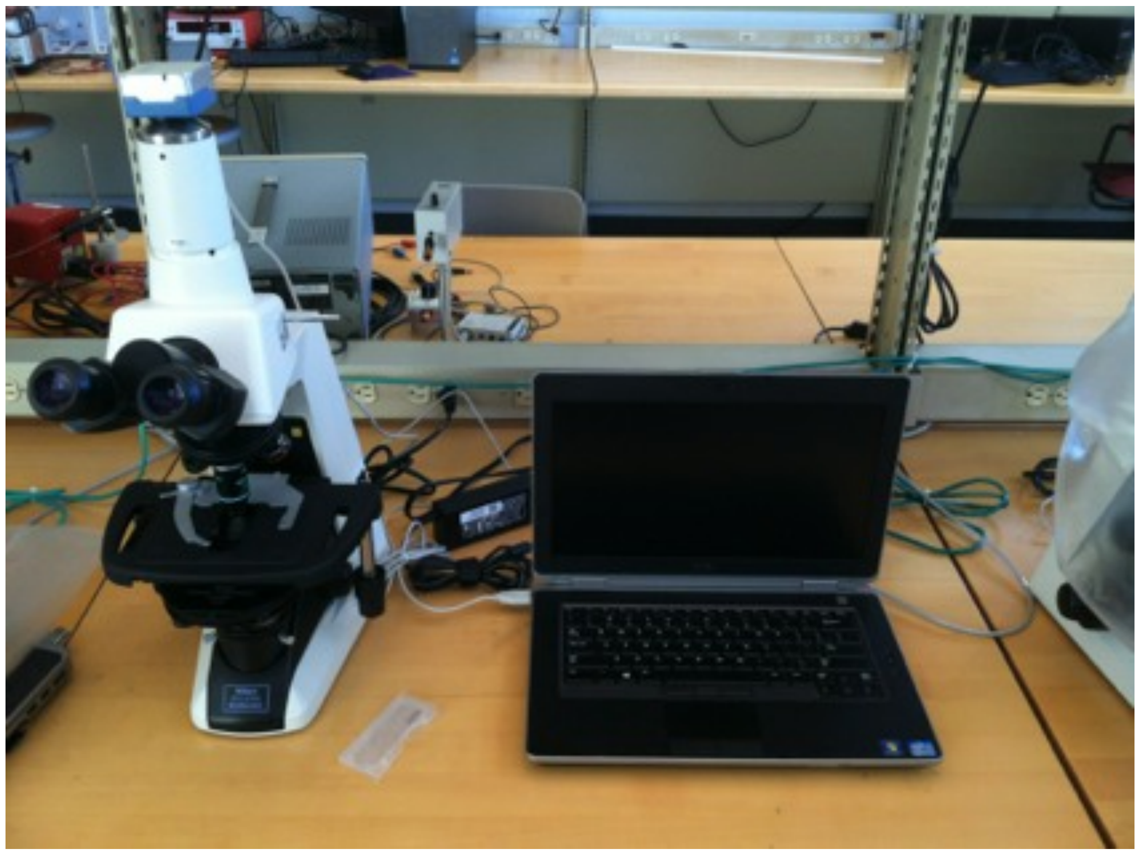
k_b : Boltzmann Constant
T : Temperature
r : Particle Radius
η : Viscosity of Medium
t : Time Step

Einstein's molecular theory of Brownian motion predicts that the mean square displacement of a particle executing Brownian motion is proportional to time.

Note: equation above is for one dimension.

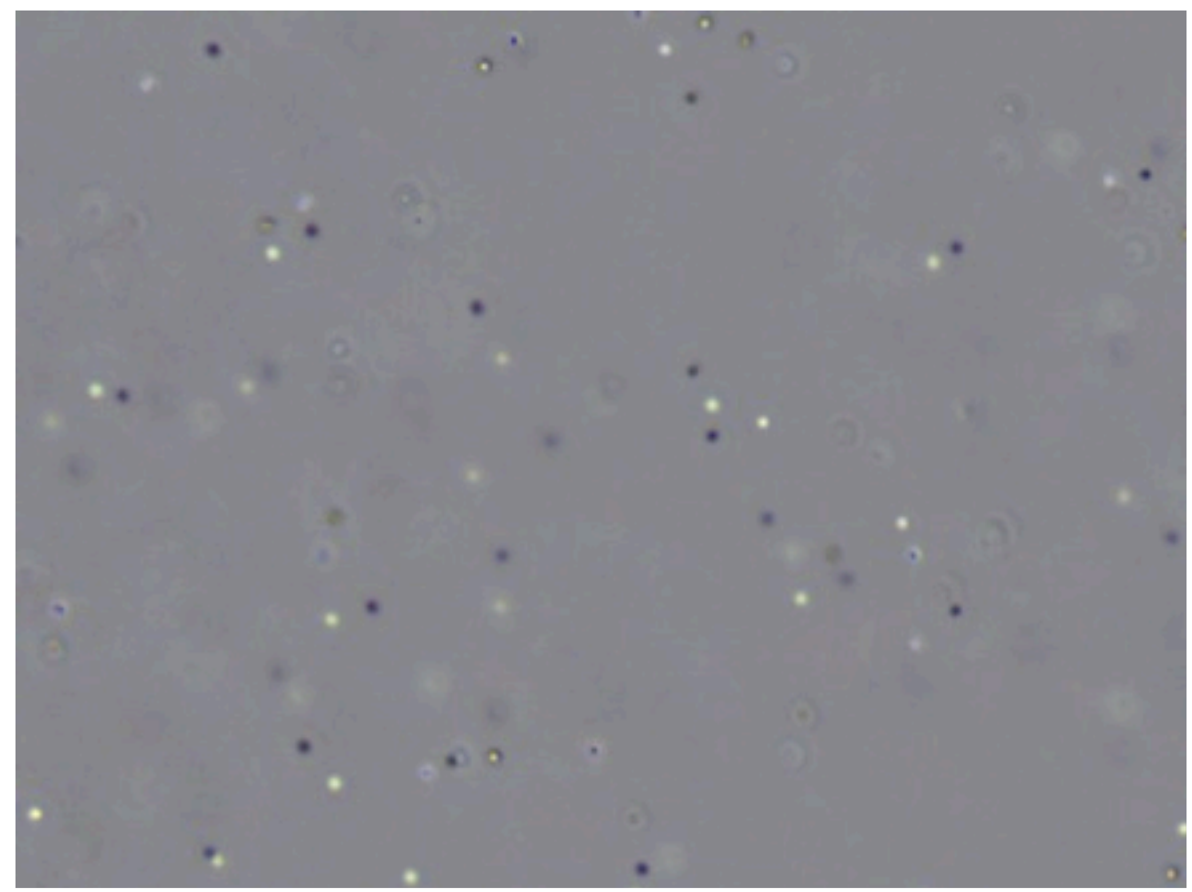
Experiment & Data

So let's measure the mean square displacements of particles as a function of time!



Use microscope+camera to obtain measurements of x & y displacements.

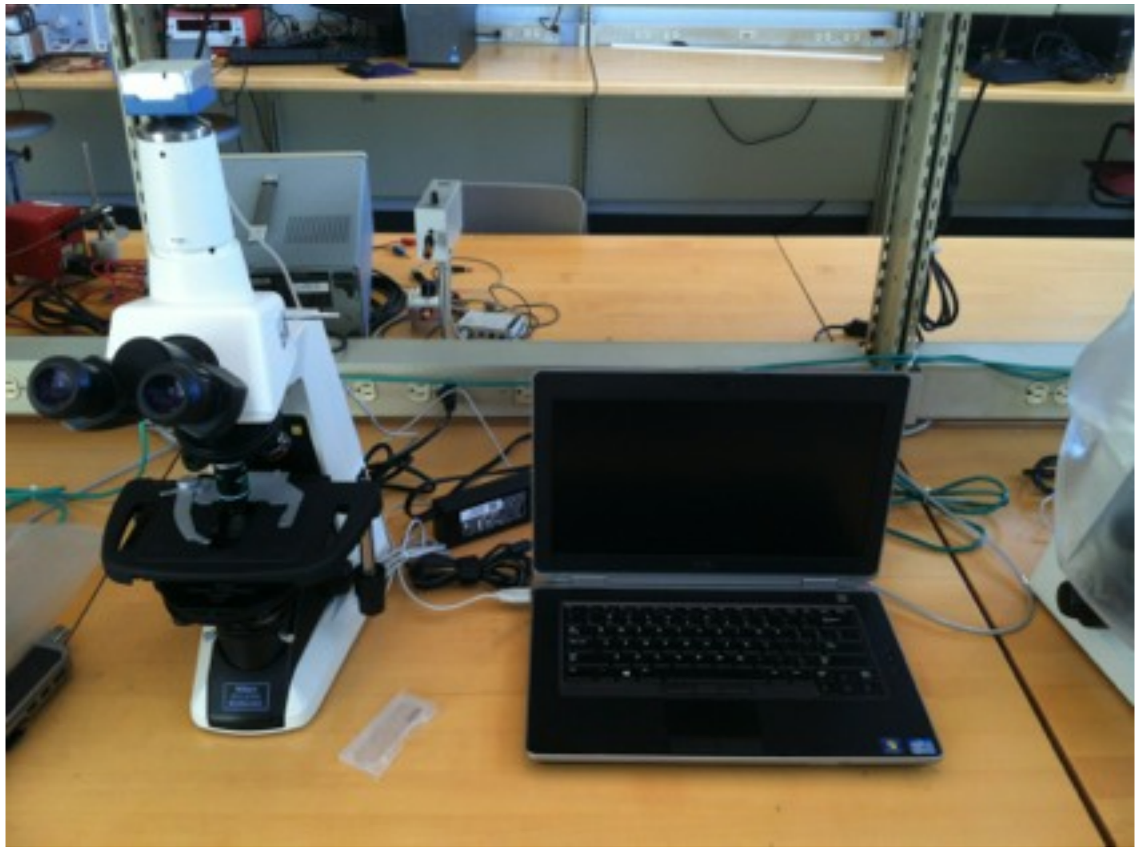
Obtain many exposures.



Measure many displacements.

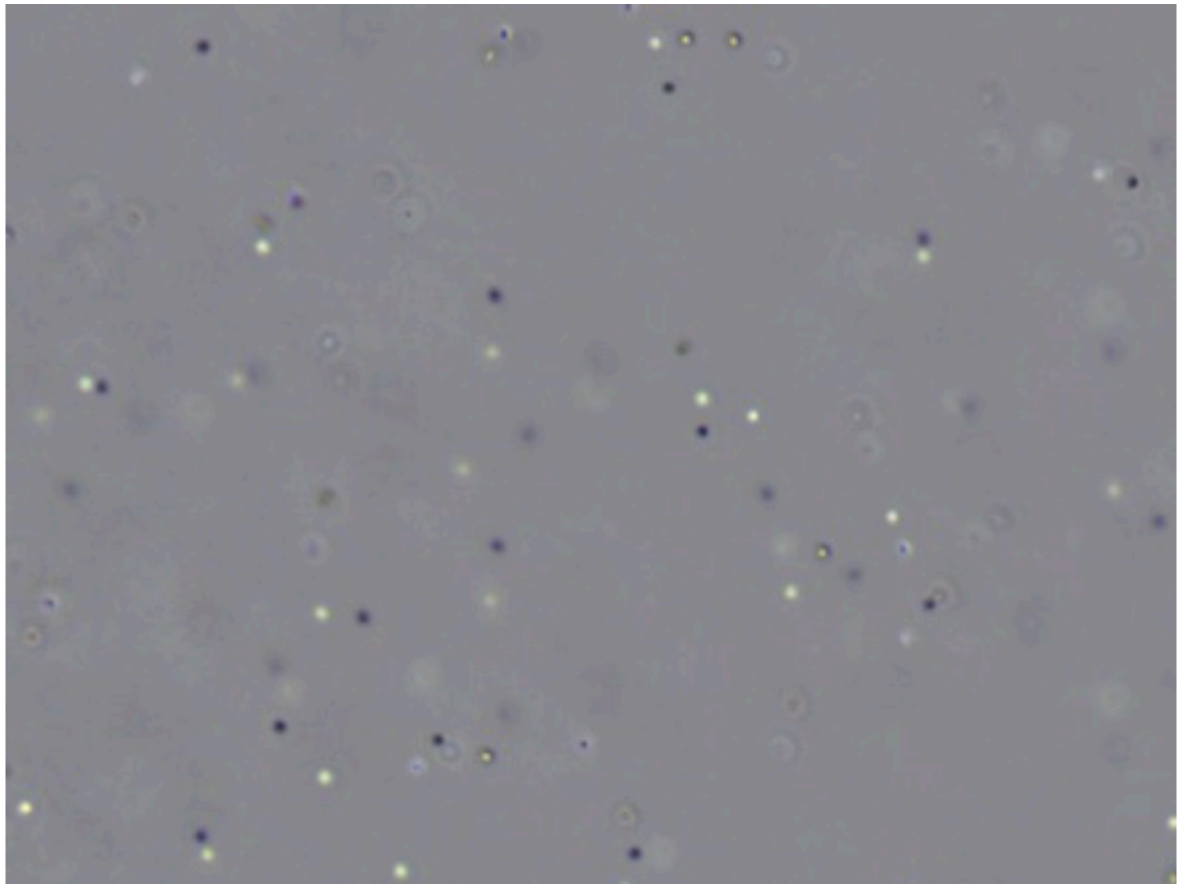
Experiment & Data

So let's measure the mean square displacements of particles as a function of time!



Use microscope+camera to obtain measurements of x & y displacements.

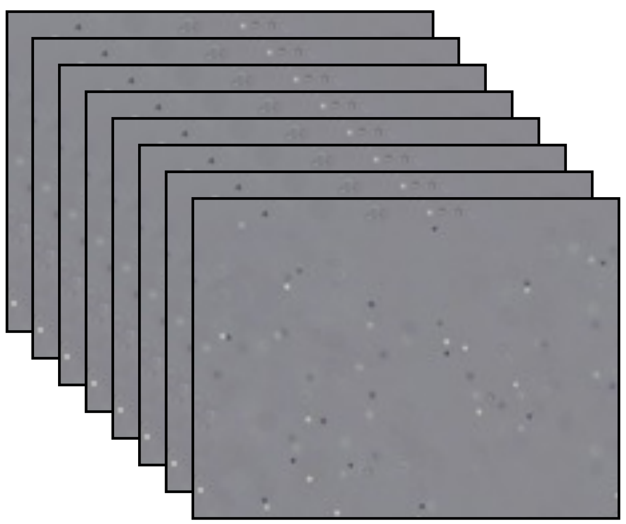
Obtain many exposures.



Measure many displacements.

Experiment & Data

Take a full "Exposure Set"



20 photos

Particle Tracking

Dataset with N=100 step measurements {x}, {y}

Don't forget to measure the temperature of the room!

Experiment & Data

Dataset with N=100 step measurements $\{x\}, \{y\}$

Subtract means (remove particle flow) $\{x-\langle x\rangle\}, \{y-\langle y\rangle\}$

Compute mean square displacement $\langle (x-\langle x\rangle)^2\rangle, \langle (y-\langle y\rangle)^2\rangle$

Repeat 5x to find sample standard deviation $\sigma_{\langle (x-\langle x\rangle)^2\rangle}, \sigma_{\langle (y-\langle y\rangle)^2\rangle}$

Compute average of results from 5 datasets

Repeat for two exposure time separations: t =2s, 3s.

Data Analysis

<u>Model</u>

$$\langle (x-\langle x\rangle)^2\rangle = \frac{k_b T t}{3\pi r\eta}$$

Express k_B as a function of measured variables

$$k_b = \frac{3\pi r\eta\langle (x-\langle x\rangle)^2\rangle}{T\ t}$$

Propagate uncertainties ($\sigma_{\langle (x-\langle x\rangle)^2\rangle}$, σ_T, σ_η) to find σ_k.

Repeat for x data, y data and two time steps. Systematics check.

Bibliography

Primary

A. Einstein, "Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen," Ann. Phys. 17, 132-148 (1905).

M. Smoluchowski, "Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen," Ann. Phys. 326, 756-780 (1906).

P. Langevin, "Sur la Theory du Mouvement Brownien," C. R. Acad. Sci. (Paris) 146, 530-533 (1908).

J. Perrin., "Le Mouvement Brownien et la Realite Moleculaire," Ann. Chimi. Phys 18, 5-114 (1909).

Pedagogical

P. Nakroshis, et al. "Measuring Boltzmann's constant using video microscopy of Brownian motion," Am. J. Phys. 71, 568-573 (2003).

R. Newburgh, J. Peidle, and W. Rueckner, "Einstein, Perrin, and the reality of atoms: 1905 revisited," Am. J. Phys. 74, 478-481 (2006).

Schedule

Week One: 1) Obtain "Exposure sets" for t=2s, 3s.
2) Upload sequence of 10 photos for discussion next Monday.
3) **Write draft report through procedure in Experiment and Data section. Submit Monday.***

Week Two: 1) Analyze images (class tutorial next Mon).
2) Upload plot/table of results for discussion on following Monday.
3) Write draft report through end of Data Analysis section. Submit following Monday.

Week Three: Finish up!

***New participation requirement.**

Schedule

Below is the nominal schedule for the course.

Date	Lecture	Weekly Reading	Other Notes
Jan 27	Class Overview, Measurement & Errors	Bev. Ch1	Work on basic LaTeX and Python examples
media:AdvLab2014Intro.pdf			
Feb 03	Probability Distributions, Exp. 1 Introduction	Bev. Ch 2	Start Exp. 1; LaTeX Tutorial
Feb 10	Propagation of Errors	Bev. Ch 3	Updates on Exp. 1; Exp. 1 Draft 1 Due; Coding Tutorial
Feb 17	Estimates of Mean and Errors	Bev. Ch 4	Updates on Exp. 1; Exp. 1 Draft 2 Due
Feb 24	Monte Carlo Techniques	Bev. Ch 5	Exp. 1 Report Due, Start Exp. 2
Mar 3	Linear Least Squares 1	Bev. Ch 6	Updates on Exp. 2; Exp. 2 Draft 1 Due
Mar 10	Linear Least Squares 2	Bev. Ch 7	Updates on Exp. 2; Exp. 2 Draft 2 Due
Mar 17-21 Spring Break			
Mar 24	Nonlinear Fitting 1	Bev. Ch 8	Exp. 2 Due, Start Exp. 3
Mar 31	Nonlinear Fitting 2	Bev. Ch 9	Updates on Exp. 3; Exp. 3 Draft 1 Due
Apr 7	Maximum Likelihood	Bev. Ch 10	Updates on Exp. 3; Exp. 3 Draft 2 Due
Apr 14	Testing the Fit	Bev. Ch 11	Exp. 3 Due, Start Exp. 4
Apr 21	TBD	-	Updates on Exp. 4; Exp. 4 Draft 1 Due
Apr 28	TBD	-	Updates on Exp. 4; Exp. 4 Draft 2 Due